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Energy-Efficiency using Critical Nodes Detection Problem in Industrial Wireless Sensor Networks (IWSNs)

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Abstract

Industrial Wireless Sensor Networks (IWSNs) play significant role in enabling self-organization, rapid deployment, and ensuring reliable monitoring and control in industrial systems. Despite their advantages, sensor nodes in IWSNs are constrained by limited energy resources, and have minimal processing power, and restricted communication capabilities, making energy efficiency is a critical concern. To solve this problem, we propose a novel approach of Critical Node Detection Problem (CNDP) based on a graph partitioning method in IWSNs. The network is divided into manageable partitions after removing critical nodes, optimizing energy usage, reducing communication overhead, and ensure the network's connectivity and operational reliability across multiple communication rounds. Experiments simulation validates our proposed approach, approving its efficiency in reducing significant energy consumption while preserving connectivity and functionality for industrial systems. Furthermore, the results highlight the potential of using critical node analysis to support sustainable and efficient operations in resource-constrained industrial environments.

Keywords: Critical Node Problem; Graph Partitioning; Reliability; Energy; Industrial Wireless Sensor Networks.

1. Introduction

The global industrial automation market is growing rapidly at 213.53 billion dollars in 2023 and can reach 529.16 billion dollars in 2033 [1]. This rise is being driven by the demand for smart manufacturing processes, in which Industrial Wireless Sensor Networks (IWSNs) play a significant role by giving solutions for real-time monitoring, control, and optimization, resulting in increased productivity and operational efficiency in industrial systems. Their specific intelligent-processing skills assist better decision-making, making them important in modern industrial environments [2].

The use of IWSNs is predicted to continue to contribute considerably to the expansion of the industrial automation market, in line with future growth patterns [3], which is limited by the unique characteristic of industrial production. It can be classified into three major applications [3], the first major application is Environmental Sensing, including detection of fire, flood, and landslide in dangerous areas; surveillance of air, water, and waste water; pollution of production materials; and security concerns in markets with rival suppliers. The second application is Condition Monitoring, covering machine condition monitoring, healthcare applications of IWSN, structure and human condition monitoring, and giving structure health information and the final application is Process Automation, which gives customers access to data on production and service resources, such as building automation, industrial processes, people, materials, stock, and supply chain status.

These networks consist of challenges in network structure and connectivity due to many factors such as humidity and vibration. Sensor nodes may fail due to these environmental factors and dynamic network topologies [4]. Additionally, IWSNs design and implementation are constrained by resources like processor, memory, and energy. Computing power of sensor nodes are constrained by their limited resources

[5]. Communication is the most energy-consuming activity, especially in industrial environments and the energy efficiency of sensor nodes is important in maintain the performance of networks [5].

In IWSN, a centralized node can be used as a control hub, which provides an efficient way to control and manage network functions. However, addressing issues such as node failure, scalability, and high energy consumption. In addition, for large networks this node may become an obstacle that affect data loads [6].

To solve such problems, many researchers focus only on detect cut-vertex known as articulation points [7], [8], [9]. A Cut vertex are nodes or link whose removal divide the network into connected components. In other words, cut vertices are crucial nodes in the network topology, they act as bridges between different parts of the network [10]. Cut vertex can ensure connectivity and reduce energy but has many limitations such as scalability and fault tolerance, practically in IWSN. Others addressed dominated set [11], [12], [13] as another technique used to minimize energy consumption by limiting the number of nodes that actively participate in communication. A dominated set is a subset of nodes within a network where each node either belongs to this subset or is adjacent to a node in the subset [14]. The nodes in the dominated set are considered to dominate their neighbouring nodes because they are responsible for performing certain tasks, such as communication. The main issues of the dominate set, it is limited in scalability, complexity, and network reliability. These limitations make dominate set method less practical in IWSNs.

Another study combines Cut Vertex detection and Boundary Nodes detection [15], as a hybrid approach to optimize the performance of IWSNs, it can improve reliability, resource utilization, and scalability for IWSNs. However, it may encounter difficulties with precise placement, static assumptions, and network situation adaptation, all of which are essential for realistic deployment in large-scale industrial environments.

The critical node detection problem is not specifically addressed in previous studies. Most researchers focus on articulation points, boundary nodes, dominate set, other researchers propose hybrid approach, by combining two different algorithms for Critical node detection, which is inefficient and increased computation and communication costs.

Critical nodes in a network are those whose failure or removal would significantly disrupt network performance [16], these nodes have an important role in efficient data transmission and maintain connectivity, while minimize energy consumption. By applying graph partitioning-based approaches and divide the network into smaller and manageable partitions, it minimizes energy-efficient for each partition. Moreover, reduce communication overhead.

The contributions of this work as follow:

1. A novel methodology for identifying critical nodes in IWSNs based on graph partitioning techniques.
2. A simulation framework to analyze the impact of critical nodes on energy efficiency and connectivity in IWSNs.
3. Insights into the trade-offs between energy consumption, network partitioning, and performance in industrial environments.

This paper is organized as follows: Section 2 reviews related work in energy efficiency, connectivity and critical nodes in IWSNs. Section 3 presents the proposed approach. Section 4 discusses the simulation and results. Finally, Section 5 concludes the work.

2. Related Works

In this section, we will review various studies that focus in solving energy consumption and connectivity in Wireless Sensor Networks (WSNs) using methods such as cut vertex detection, dominating set, and critical node identification, which enhance network efficiency, reliability, and network lifespan. The cut vertex method is widely used by many researchers such as in [7], they select the most important nodes in the network, to improve security and energy use, make sure the energy-efficient link monitoring strategy focuses on the appropriate areas. Authors [17], [18] in used cut vertex in connectivity estimation ensures that the network can continue to communicate even if some nodes fail or if the network topology, they improve network fault tolerance and reliability. Another research [19] also used cut vertex that ascertain k-connectivity even if the node fails, so they detect and protect some special nodes and adding non-critical nodes to cover the gaps. By locating and controlling cut vertices, the network still connected and avoids disconnection, ensuring optimal operation by dynamically adjusting the network topology to compensate for energy depletion. Others [20] based on the dominating set, to optimize coverage, minimizes energy consumption, and improves energy efficiency by selecting a small subset of nodes for full connectivity. In [21] they combine Minimum Connected Dominating Set (MCDS) and a bi-partite inspired technique to effectively select cluster head selection, they optimize the energy consumption of the network while preserving connectivity and performance. Authors in [22] suggest a protocol that used centralized nodes to ensure effective energy consumption, network performance optimization, network lifespan extension, and connectivity enhancement. In [23] also based on centralized node to optimize routing and energy consumption. Others [24] based on boundary nodes, which are crucial in optimizing coverage and energy efficiency in sensor network, as they define coverage regions in the Voronoi diagram. In [25] they based on these nodes to optimize energy efficiency sensor network designs. In order to identify and report events at the perimeter, boundary nodes are positioned strategically at the cluster edges. Their proposed protocol can effectively reduce energy consumption and controlling data transmission, particularly in large-scale networks. Authors in [15] consider critical nodes, they combine Cut Vertex and Boundary Nodes detection, to improve reliability, resource utilization, and scalability for IWSNs. All this work doesn’t really consider Critical Nodes Detection problem, just combine two algorithm and considerate as critical nodes. In the context of Industrial Wireless Sensor Networks (IWSNs), their specific requirements such as high reliability, energy efficiency, and robustness, deal with difficult situations frequently like complex topologies. Therefore, selecting the appropriate method for identifying critical nodes has a significant impact on the performance of IWSNs. However, the previous mentioned methods have drawbacks, including high computational complexity, and the need for dynamic updates in large or mobile networks. Our proposed approach based on identifying Critical Nodes, which is more effective strategy for improving network performance, energy efficiency, and reliability in real-time industrial applications. In the following section we detail our methodology.

3. Contribution

In this section, we explain our novel proposed methodology to solve the problem of energy consumption, and ensuring connectivity in Industrial Wireless Sensor Networks (IWSNs). The proposed approach is based on the concept of critical node detection, in order to optimize network performance by selecting some specific nodes regarding on their importance in the network’s structure.

Instead of traditional methods, including cut vertices, dominating sets, boundary nodes, and centralized nodes, we propose a novel method considering Critical Node Detection Problem, in order to enhance energy efficiency and connectivity. We take into consideration graph theory concepts, covering connected component size and Maximal Independent Sets (MIS) to identify these nodes more effectively. Our novel approach includes three phases:

3.1 Critical Node Detection

The Critical Node Problem aims to identifying important nodes that have a key role for maintaining network connectivity and ensuring efficient data flow in IWSNs, based on network’s structure. Using a graph theory concept, when the removing these nodes, it can significantly affect connectivity [16] . These nodes are selected by building a Maximal Independent Set (MIS) in order to make an efficient strategy of partitioning. It selects nodes in such a way that no two nodes in the MIS are adjacent, reducing communication interference between them, then determine if the network will disconnect if the identified critical nodes are removed, then verify each resulting connected component size. In another words, the nodes that, if removed, break-down the network into separated partitions, this is known as critical nodes. This guarantees that the nodes chosen are essential to preserving the network's connectivity.

The key challenge addressed by our proposed methodology is the selection of nodes that have a high importance in maintaining the network’s connectivity while also minimizing energy consumption. To tackle this problem, we adopt more enhanced approach that based on both the size of connected components and Maximal Independent Sets (MIS).

3.2 Graph Partitioning

After identifying and removing critical nodes from the network, the latter is partitioned into different partitions these partitions are generally connected components, which can significantly decrease communication overhead by limiting interactions between distant nodes and also preserving energy.

Critical nodes are identified based on their importance in maintaining network connectivity and reducing interference. These nodes frequently serve a critical role in ensuring that communication between nodes within the network. Once found, these nodes are removed, and their absence is used to determine how the network responds.

After removing these nodes, the network is divided into connected components. A connected component is a network subgraph in which all nodes are reachable from one another but no node within the component is connected to any node outside of it [26]. The degree to which the network is divided into separate components gives useful information about the significance of the eliminated nodes. If the network divides into a significant number of components after eliminating a few nodes, it indicates that these nodes were actually critical for maintaining overall connectivity, hence lowering communication overhead. This occurs because nodes within the same component communicate more frequently, whereas interactions between nodes in other components are reduced. In addition, it limits interactions between distant nodes, and that decreases the requirement for long-distance communication, it requiring less energy for data transfer. This is especially useful in energy-constrained context.

Node in smaller components are close to each other's, the data transmission is more localized, which results in shorter communication flows that need less power. So, improving energy conservation, and the effectiveness of the network. If the number of components does not increase, this means that removed nodes are not critical to the network's structure and connectivity. Our objective is to reduce energy consumption, and ensure connectivity of the network. When we achieve a balance between minimizing communication overhead and avoiding severe network fragmentation, to enhance energy efficiency the partition of network into smaller components, the nodes within the same component can continue to communicate efficiently, without the need for long-range communication that consumes more energy. Additionally, reduce communication between components, can minimize the overall energy spent on data transmission.

3.3 Energy Consumption

The removal of critical nodes reduces unnecessary energy consumption by minimizing redundant communication paths. Energy consumption is calculated iteratively, and the results are analysed to ensure that the proposed approach improves energy efficiency without compromising network connectivity. The energy used by each node is calculated based on its CPU time usage, which is a simplified model for energy consumption.

In Algorithm 1, we present three phases as shown in Figure 1, begin by identifying Critical Node based on their role in network connectivity using methods of graph theory like connected component analysis and MIS, then we remove these critical nodes from the network. After that the graph partitioning phase, to determine the connected components of the remaining network. If the number of components increases significantly, this is mean that the removed nodes are critical sure. Moreover, the constraint size of each connected component is less than or equal to a predefined threshold L, and that ensure the network fragmentation (no component becomes too large). Otherwise, If the size of components is more than L, reviewing the choice on which nodes are critical. Additional nodes may need to be changes made to ensure effective connectivity.

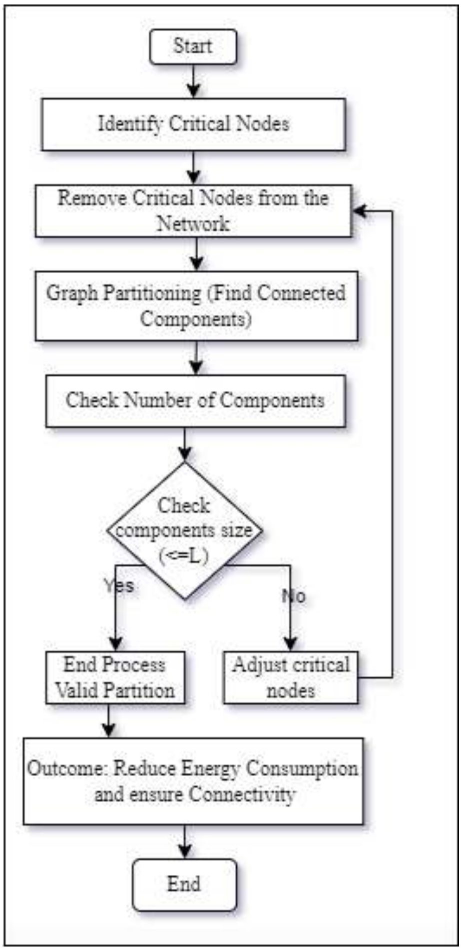

Figure 1: Flow chart of the proposed approach

Algorithm 1: Critical Node-Based Energy Optimization in IWSNs (CNEO)

```
Input: G(V, E)
L: size of connected components after partitioning.
E_initial: Initial energy of each node.
Iterations: Number of iterations for averaging results.
Output:
E: Total energy consumption after removing critical nodes.
C: Number of connected components after partitioning.
Phase 1: Detect Critical Nodes
1. Initialize MIS (Maximal Independent Set) as an empty set.
2. Shuffle nodes randomly.
3. For each node v in G:
If v is not in Visited:
Add v to MIS.
Mark v and its neighbours as visited.
4. Compute Removed Nodes = Nodes(G) - MIS.
5. Create subgraph G_sub using MIS.
6. For each node u in Removed Nodes: Add u to G_sub and connect it to its neighbours. If all connected components in G_sub are ≤ L, retain u in G_sub.
7. Return Critical_Nodes = Nodes(G) - Nodes(G_sub).
Phase 2: Partition Graph
1. Remove Critical_Nodes from G to create G_reduced.
2. Compute the number of connected components in G_reduced.
3. Record component sizes and the number of components.
Phase 3: Calculate Energy Consumption
1. For each node v in G,
   set Energy[v] = E_initial.
2. For each iteration:
For each node v in G:
If Energy[v] > 0:
 Randomly select T_CPU.
 Compute energy consumed:
 E_CPU = P_CPU * T_CPU.
3.Sum the remaining energy across all nodes.
Return energy consumption, number of partitions
```

4 Results

Our proposed methodology CNEO (Critical Node Energy Optimization) is compared with CDSCUT and D-LPCN [27], ABCND [15], and ABIDE and D-LPCN [8].

IWSN nodes were randomly placed inside the networks. We have chosen a single sink node to investigate the network settings.

We used NetworkX and Python to implement the simulation. In order to examine the power consumption, the nodes are progressively expanded within a 1000 × 1000 m2 area. Fifty milliseconds is the transmission range.

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The evaluation and comparison criteria, including energy consumption, and the percentage of correctly detected critical nodes.

The simulation of network is run over the 20 iterations.

4.1 Power Consumption as Network Nodes Increase

One important factor to consider in WSNs is power consumption. Our algorithm is compared against ABCND, ABIDE and D-LPCN, and E-CDSCUT and D-LPCN algorithms for various numbers of nodes (See Figure 2). For all methods, the resulting Fig. demonstrates that power usage increases with the number of nodes. When there are more nodes, we have more communication between these nodes, this results to higher energy consumption. Our CNEO algorithm uses nearly less energy than other algorithms, and less than the ABCND algorithms. So, our proposed algorithm CNEO can optimize the energy consumption of the network.

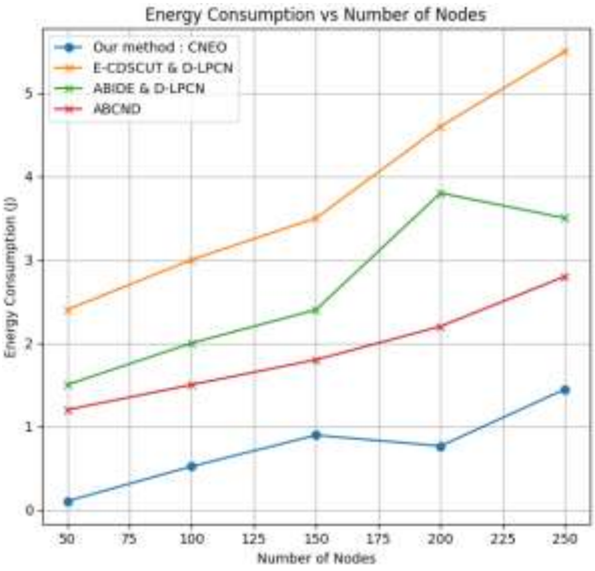

Figure 2: Comparison of energy consumption vs number of nodes

In this study, we evaluate the impact of removing critical nodes in a IWSNs. It simulates energy consumption in the network.

The energy consumption generally increases as the number of nodes increase. More nodes signify more communication and higher total energy usage.

The energy consumption after removing critical nodes is compared with other methods. As we seen that our proposed Method: CNEO shows lower energy consumption compared to other methods, indicating its effectiveness in minimizing energy. The energy can be reduced when critical nodes are removed, it minimizes communication and energy in the network.

4.2 Number of Partitions as Network Node Increase

Removing critical nodes results in partitioning the network into different partitions, which are smaller connected components, to ensure the fragmentation of the network.

Our proposed algorithm can effectively partitioned network based on the Maximal Independent Set (MIS) approach and the size constraint L [28].

After comparing with other methods (e.g., E-CDSCUT and D-LPCN, ABIDE and D-LPCN, ABCND), as we seen from Figure 3, they consume more energy than our method: CNEO, which mean it shows better energy efficiency.

The proposed method can have a good balance between achieving good results in minimizing the energy consumption and fragmentation of the network.

By strategically identifying and removing critical nodes. It partitions the network into manageable components, which could be useful in many applications' domain.

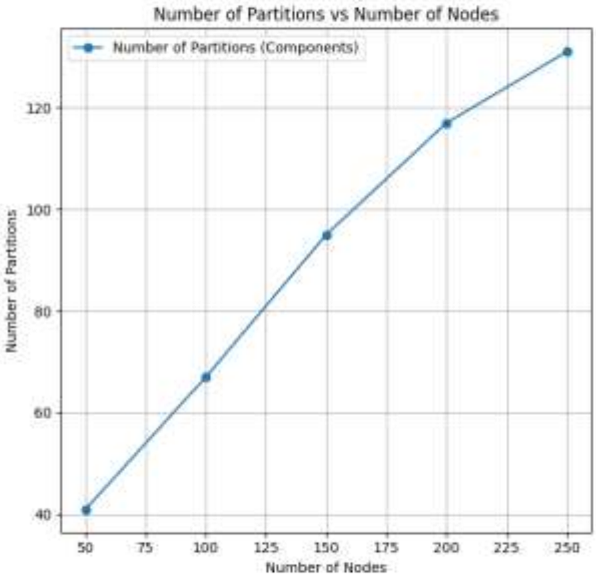

Figure 3: Number of partitions after removing critical nodes vs number of nodes

5 Conclusion

This study focuses on improving the energy efficiency and structural optimization of Industrial Wireless Sensor Networks (IWSNs). This work achieves a balance between energy consumption and connectivity by leveraging the identification and removal of critical nodes using the proposed method Node Energy Optimization (CNEO). The proposed method greatly reduces energy consumption by tragically deleting critical nodes, resulting in low energy usage while retaining functional partitions.

The use of graph-based approaches, such as the Maximal Independent Set (MIS), guarantees that the network is effectively partitioned into smaller, more controllable components, improving fault tolerance.

The method is adaptive to different networks, with consistent performance and scalability. This is ideal for a large wide of applications such as Internet of Things, environmental monitoring, and disaster recovery networks.

Future research can be realized by extending the algorithm to handle dynamic topologies, evaluate the method in real-world WSN to address different challenges, and enhancing the approach to balance energy consumption with prolonged network lifetimes.

Conflicts of interest

The authors declare no conflict of interest.

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