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Nonlinear Behavior of a Non-Homogeneous Plate in Cylindrical Bending under Uniform Loading

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Abstract

In this work, we will study the non-linear behavior of a plate in cylindrical bending using an exponential function with gradient of material properties (Commonly called E-FG). The plates are subjected to uniform loading and geometric nonlinearity is introduced into relationship the stress-strain using the expressions nonlinear deformations of Von Karman's. The material properties of the plate, except the Poisson coefficient, are assumed to vary in the direction of thickness z in the form of an exponential law distribution. The solution is obtained by using the Prince of Hamilton. Numerical results by an exponential function with gradient of properties are given in the form of graphs non-dimensional; and determine the effect of the material properties on the deflection and the normal stress across the thickness.

Keywords: Exponential function with gradient of properties (E-FG); nonlinear behavior; deformation; plate.

1. Introduction

The composite material consists of the assembly of two or more materials of different nature, which makes it possible to increase the required performance. However, the discontinuity of the material properties through the interface of the composite material constituents causes stress concentrations under mechanical and thermal loads. To eliminate singular stresses in an ultra-warm environment, the concept of materials with gradient properties (FGM) was introduced in 1984 by a group of scientists in Japan [1, 2].

Functionally graded materials (FGMs) are under the microscope of non-homogeneous composite materials, their mechanical properties vary gradually and continuously from one surface to another. The composition changes from a ceramic surface to a metal surface following to a function of the volume fraction of the two materials between the two surfaces.

FGMs plates are generally used in thin structures and therefore, it is interesting to study and understand the non-linear behavior of plates with gradient materials under uniform loading. Several linear studies of the flexural FGM plates in a thermal environment are presented [3–8]. However, the investigations in nonlinear analysis of the FGM plates under thermal or mechanical loading are limited in number. For example, Praveen and Reddy [9] have analyzed the non-linear response of ceramic-metal gradient material plates using the finite element method taking into account transverse shear deformations, Polar inertia and large moderate rotations in the sense of Von Karman. Reddy [10] presented the solutions of rectangular plates in FGM using the third order theory of plate shear deformations. The large deformations of the FGM plates under uniform loading were also studied by GhannadPour and Alinia using the Von-Karman theory [11]. Through the thickness, the distribution of the stresses of the aluminum and alumina plates is linear in contrast in the plates in FGM the behavior is nonlinear and is a function of the variation of the properties in the direction of the thickness. A similar method was used by Sun and Chin [12, 13], Navazi *et al.*[14] Concerning the non-linear cylindrical flexion analysis of plates based on classical theory (CPT).

The objective of our research is to determine the displacements and the stresses of the plates E-FG[1] in cylindrical bending under uniform loading. The equilibrium equations are obtained on the basis of the

[1] Exponential function with gradient of material properties (Commonly called E-FG)

classical theory of plates. Using non-linear deformations of Von Karman and the gradual variation of material properties, the non-linear equilibrium equations are obtained and are thus reduced to a linear differential equation. This equation is solved by the boundary conditions of a simply supported plate and we will study the effect of several parameters such as the index of the volume fraction, the type of loading and the dimensions of the plate.

2. Theory and Formulation

2.1. Properties material of the E-FGM plate

In this study, we consider a rectangular elastic plate E-FGM with uniform thickness h and a length $l=2a$. The plate is made of a mixture of ceramic-metal; and its composition is assumed to be gradual varies from the top to the bottom surface. In fact, the top surface ($z = h/2$) of the plate is ceramic-rich whereas the bottom surface ($z = -h/2$) is metal-rich. Consequently, the modulus of elasticity is a function of z, measured from the medium plane of the plate. There are several models analytical and mathematics to select the proper function of material properties of the FGM. These functions are supposed to be simple and continuous, and may have concave and convex curvatures [15]. In this study, an exponential function to describe the material properties of the FGMs is chosen. The relation between E and z of the FGM ceramic-metal plate is given by the equation below, expressed by Sallai *et al* [16]:

$$E(z)=\mathrm{A}e^{B(z+h/2)} \tag{1}$$

with

$$A = E_2 \text{ and } B = \frac{1}{h}\ln\left(\frac{E_1}{E_2}\right) \tag{2}$$

where $E(z)$ indicates the Young's modulus, E_1 and E_2 expresses respectively the Young's modulus of the upper surface ($z=+h/2$) and lower ($z=-h/2$).

2.2. Nonlinear equations of E-FGM plates in cylindrical bending

The fundamental equations of a large deformation analysis of an FGM plate subjected to uniform loading are briefly presented in this section. The use of classical plate theory (CPT) assumes that Kirchhoff's hypotheses are united. Kirchhoff's hypotheses assume that the displacements are of the form:

$$u(x,y,z) = u_0(x,y) - z\frac{\partial w_0}{\partial x} \tag{3}$$

$$(x,y,z)=0 \tag{4}$$

$$w(x,y,z) = w_0(x,y) \tag{5}$$

where (u,v,w) are respectively the displacements in the directions (x,y,z). Also, (u_0,v_0,w_0) are respectively the displacements of the medium plane in the same directions.

The nonlinear deformation-displacement relations of Von Karman are as follows:

$$\varepsilon_x = \frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2 - z\frac{\partial^2 w_0}{\partial x^2}, \tag{6}$$

$$\varepsilon_y = \varepsilon_z = 0, \tag{7}$$

$$\gamma_{xy} = \left(\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}\right) - 2z\frac{\partial^2 w_0}{\partial x \partial y} \tag{8}$$
$$\gamma_{xz} = \gamma_{yz} = 0,$$

The law of constraint-deformation behavior is expressed in the form:

$$\begin{Bmatrix}\sigma_x\\ \sigma_x\\ \tau_{xy}\end{Bmatrix} = \begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_x\\ \gamma_{xy}\end{Bmatrix}, \tag{9}$$

Using the materials properties given by eq. (1), the stiffness coefficients Q_{ij} can be expressed by:

$$Q_{11} = Q_{22} = \frac{E(z)}{1-\nu^2}, \tag{10}$$

$$Q_{12} = \frac{\nu\, E(z)}{1-\nu^2}, \tag{11}$$

$$Q_{66} = \frac{E(z)}{2(1+\nu)}, \tag{12}$$

Using the Hamilton Principle, the governing equations in the description Euler - Lagrange are:

$$N_{x,x} = 0, \tag{13}$$

$$Q_{x,x} + q + N_x w_{,xx} = 0 \quad (14)$$

$$M_{x,x} - Q_x = 0 \quad (15)$$

where q is the transverse loading. The normal force, the shear force and the moment results are given by:

$$(N_x, Q_x) = \int_{-h/2}^{h/2} (\sigma_x, \tau_{xy}) dz \quad \text{and} \quad M_x = \int_{-h/2}^{h/2} \sigma_x z \, dz \quad (16)$$

From the eq. 13 we obtain:

$$N_x = N_x^0 = const. \quad (17)$$

Therefore, Eq. 14 and 15 become

$$M_{x,xx} + q + N_x^0 w_{,xx} = 0, \quad (18)$$

By substituting Eq. 6, 7 and 8 in Eq. 9 and replace the result in Eq. 16, the resulting forces as a function of the components of the displacements can be presented as follows:

$$N_x = A_{11}\left(u_{,x} + \frac{1}{2} w_{,x}^2\right) - B_{11} w_{,xx}, \quad (19)$$

$$M_x = B_{11}\left(u_{,x} + \frac{1}{2} w_{,x}^2\right) - D_{11} w_{,xx}, \quad (20)$$

A_{11}, B_{11} and D_{11} are called the membrane stiffness, the bending coupling stiffness and the bending stiffness, respectively and are defined as follows:

$$A_{11} = \int_{-h/2}^{h/2} Q_{11} dz$$
$$B_{11} = \int_{-h/2}^{h/2} Q_{11} z dz \quad (21)$$
$$D_{11} = \int_{-h/2}^{h/2} Q_{11} z^2 dz$$

By substituting Eq. (20) in Eq. (21) we obtain

$$M_x = \frac{B_{11}}{A_{11}} N_x^0 + \left(\frac{B_{11}^2}{A_{11}} - D_{11}\right) w_{,xx} \quad (22)$$

By substituting Eq. (22) in Eq. (18), we obtain

$$W_{xxx} - k^2 w_{,xx} = q_0 \quad (23)$$

or

$$k^2 = \frac{N_x^0}{\left(D_{11} - \frac{B_{11}^2}{A_{11}}\right)}, \quad (24)$$

$$q_0 = \frac{q}{\left(D_{11} - \frac{B_{11}^2}{A_{11}}\right)}, \quad (25)$$

3. The general solution

In this study, we assume a plate E-FG submitted to a uniform transverse loading q in its upper surface. It is intended herein to determine the analytical solution of a plate E-FG in non-linear bending.

3.1. Nonlinear analysis

Eq. (24) is a fourth order differential equation. The general solution:

$$w(x) = C_1 \cosh(kx) + C_2 - \frac{q_0}{2k^2} x^2 \quad (26)$$

The C_1 and C_2 constants can be determined using the boundary conditions at the extremities of the plate. Suppose that the origin of the coordinate system is situated in the middle of the plate, the boundary conditions are:

$$w(a) = w(-a) = 0 \quad (27)$$

$$M_x(a) = M_x(-a) = 0 \quad (28)$$

$$u(a) = u(-a) = 0 \quad (29)$$

Since N_x is an unknown constant along the x-axis, the displacement in plan u can be achieved by integrating Eq. (13) depending on the length of the plate, using the general solution shown in Eq. (26). The boundary conditions can be expressed as:

$$w(a) = 0 \quad (30)$$

$$M_x(a) = \frac{B_{11}}{A_{11}} N_x^0 - \left(D_{11} - \frac{B_{11}^2}{A_{11}}\right) w_{,xx}\Big|_{x=a} = 0 \quad (31)$$

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$$u(a)=\int_0^a\left(\frac{N_x^0}{A_{11}}+\frac{B_{11}}{A_{11}}w_{,xx}-\frac{1}{2}w_{,x}^2\right)dx=0 \tag{32}$$

By substituting Eq. (26) in Eq. (30), (31) and (32) and evaluate the integral of Eq. (32):

$$C_1=\left(\frac{B_{11}}{A_{11}}+\frac{q_0}{k^4}\right)\frac{1}{\cosh(ka)} \tag{33}$$

$$C_2=\frac{q_0a^2}{2k^2}-C_1\cosh(ka) \tag{34}$$

$$\begin{aligned}u(a)=&\frac{N_x^0}{A_{11}}a+\frac{B_{11}}{A_{11}}kC_1\sinh(ka)-\frac{B_{11}}{A_{11}}\frac{q_0}{k^2}a\\&-\frac{1}{2}k^2C_1^2\left(\frac{1}{4k}\sinh(2ka)-\frac{a}{2}\right)\\&-\frac{q_0^2a^3}{6k^4}+C_1\frac{q_0}{k^3}\left[ka\cosh(ka)-\sinh(ka)\right]=0\end{aligned} \tag{35}$$

These three equations contain three unknown quantities: C_1, C_2, N_x^0 and a numerical method are used to obtain their solutions.

3.2. Linear analysis

Consider the theory of small deformations, the infinitesimal deformations are applicable and the nonlinear term of the deformations of Von Karman is neglected. By neglecting the nonlinear terms of the equilibrium equations *i.e.* the second term of Eq. (24), the following solution is obtained for a linear analysis:

$$w(x)=\frac{1}{24}q_0\left(x^4-a^4\right)x^3+\frac{1}{12}\left(\frac{2B_{11}^2-3D_{11}A_{11}}{D_{11}A_{11}}\right)q_0a^2\left(x^4-a^4\right) \tag{36}$$

$$u(x)==\frac{1}{6}q_0x^3\frac{B_{11}}{A_{11}}-\frac{1}{6}q_0a^2x\frac{B_{11}}{A_{11}} \tag{37}$$

4. Numerical application and discussion

We assume that the Young's modulus of the upper surface of the plate E-FGM, E2, is 70 GPa, and that of lower surface E_1 varies with their ratio (E_1/E_2). Note that the Poisson's coefficient is constant and equal to 0.3 for the two constituents. The dimensions of the plate are h=5 mm and a = 0.5 m. The results obtained from the analysis are presented in non-dimensional terms as follows:

- Length $\bar{x}=\dfrac{x}{a}$;
- coordinate thickness $\bar{z}=\dfrac{z}{h}$;
- deflection $\bar{w}=\dfrac{w}{h}$;
- axial stress $\bar{\sigma}=\dfrac{\sigma_x}{Q_{11m}}\left(\dfrac{h}{a}\right)^2$

  where qui Q_{11m} is the coefficient of stiffness of the metal plate,
- load parameter $q^n=\dfrac{q_0a^4}{E_mh^4}$

Figure 1 shows the variation of the Young's modulus of the plate E-FGM as a function of the non-dimensional thickness of the plate for E_1/E_2 variable.

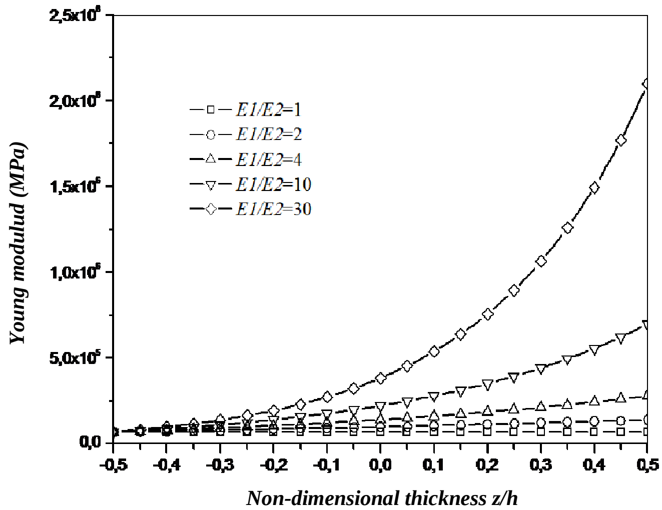

Fig. 1: The variation of the Young's module of an E-FG plate for different ratio of E_1/E_2.

Figure 2 shows the variation of the maximum deflection of the plate E-FG with, for example, E_1=380 GPa and E_2=70 GPa depending on the load parameter $\bar{q}$. It shows that for maximum deflection higher than 0.25 h the nonlinear solution is necessary. The increase in the intensity of the load generates smaller deflection in nonlinear analysis than those found in linear analysis. This type of behaviour is already covered in the literature on anti symmetric compounds [18].

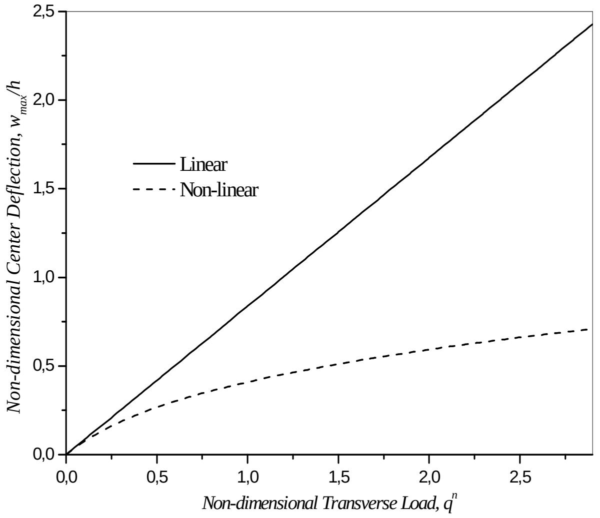

Fig. 2: Variation of the non-dimensional center deflection w_{max} of the E-FG plate versus q^n.

Figure 3 and 4 illustrates the variation of the non-dimensional deflection as a function of the non-dimensional deflection for different ratio E_1/E_2 in linear and non-linear analysis respectively. The E-FG plate is subjected to load $q = 1$ KN/m². The linear solution overestimates the deflection of the plate E-FG.

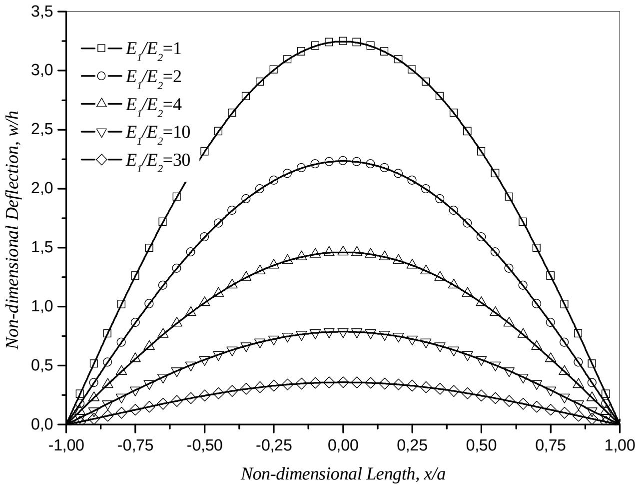

Fig. 4: Non-dimensional deflections due to transverse load $q = 1$ KN/m² versus non-dimensional length for different E_1/E_2 in non-linear analysis.

Figure 5 illustrates the non-dimensional variation of the maximum deflection of the E-FG plate with different values of E_1/E_2 subjected to uniform transverse loading.

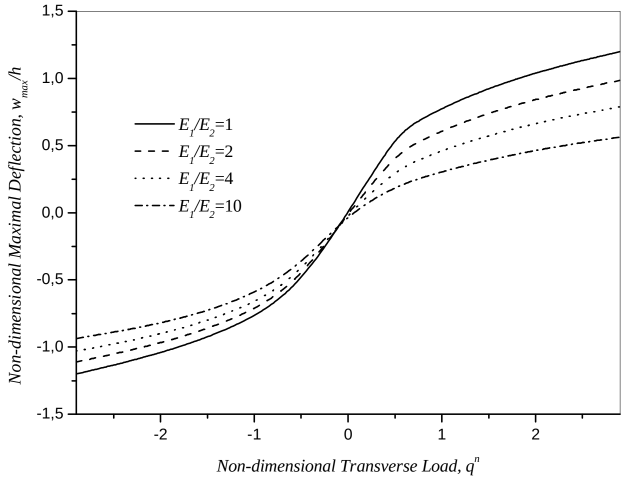

Fig. 5: Variation of the non-dimensional center deflection w_{max} of the E-FG plate versus q^n for different E_1/E_2

The result shows that the homogeneous plate (E_1/E_2=1) has a larger deflection. It also shows that the plate has a different behaviour under a positive and negative transverse loading. Under negative loading, at the beginning of loading, nonlinear analysis shows large deflection. However, under positive loading, we note an important effect of the ratio E_1/E_2.

Figure 6 and 7 show the distribution of non dimensional stress σ_x as a function of the thickness of the plate E-FG subject to uniform loading $q = 1$ KN/m² for different E_1/E_2 in linear and nonlinear analysis, respectively. Under uniform loading, compressive stresses appear at the lower fiber and tensile stress at the upper fiber. In the linear case, it can be seen that for a homogeneous plate for E_1/E_2=1, the value of the tensile and compression stresses are equal. However, for the nonlinear case, this observation is not verified. The stress for a homogeneous plate varies linearly across the thickness for linear and nonlinear analysis. The study of these figures points out that when the ratio E_1/E_2=1 increases, the intensity of the tensile and compression stresses is not equal.

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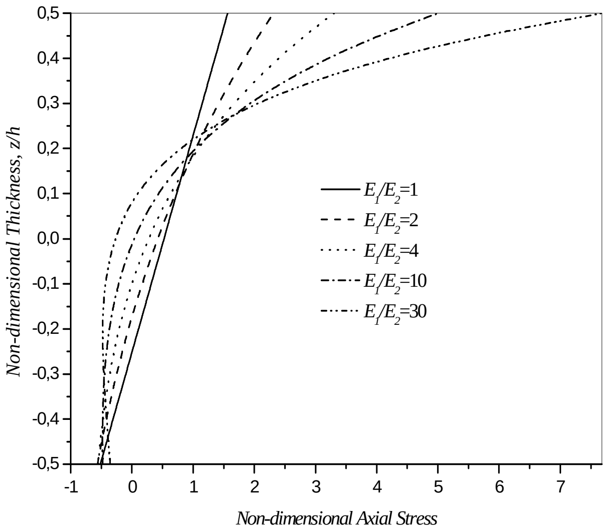

Fig. 7: Through the thickness distribution of non-dimensional axial stress $\bar{\sigma}_x$ of the E-FG plate subjected to q = 1 KN/m² for different E_1/E_2=1 in non-linear analysis.

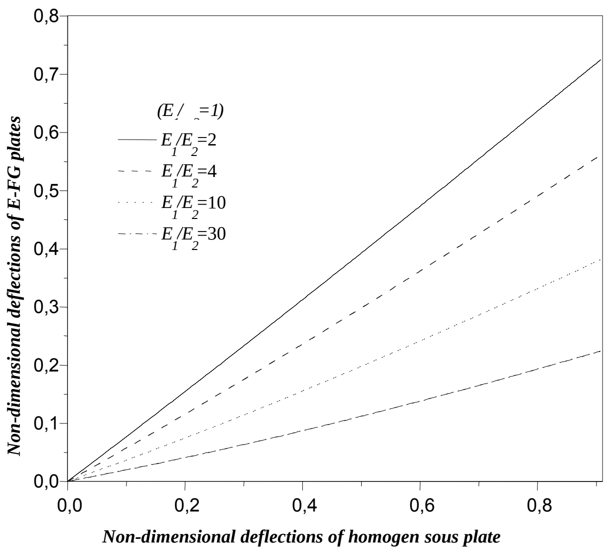

Fig. 8: Non-dimensional deflections of E-FG plates versus non-dimensional deflections of homogeneous plate for various material parameters in non-linear analysis (q = 1 KN/m²).

5. Conclusion

The nonlinear analysis of the E-FG plate in cylindrical bending under uniform loading is studied. The fundamental equations for an E-FG thin plate are obtained using the Von-Karman theory of large deformations. The material properties of E-FG plates are assumed to vary across the thickness of the plate.

For the problem in cylindrical bending, we have find that the Navier equations under the theory of large deformations can be expressed in linear equations of the deflection using non-linear boundary conditions. This linearity of the differential equations simplifies the analysis of the large deformations. The stresses and deflection are calculated for plates with a ceramic-metal mixture. The numerical results show that the nonlinear effects of the plate responses are significant. Otherwise, the results indicate that the nonlinear effect increases the intensity of transverse deflection.

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